An Example of Calculus Function: Height and Age

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Math 108: Introduction to Calculus

My motto for teaching Math 108: ***Math Is Easy, Fun, and Everywhere***.
Function and derivative are two fundamental concepts in mathematics. The following figure is used as one of the examples to help students understand the two important concepts:

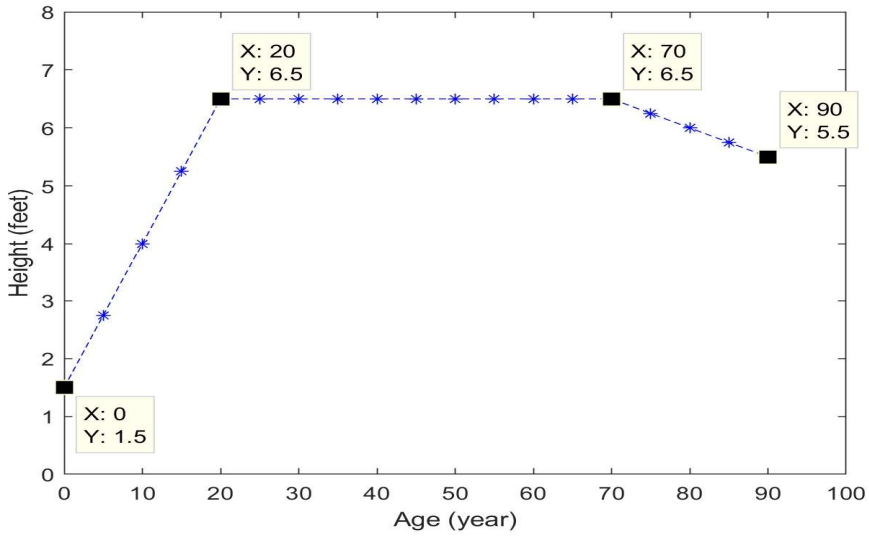

Figure 1: A person's height at different ages

This figure is used in three different lectures:

* First lecture, Section 1.1 -- Functions:

Use the information provided in the figure to answer the following questions:

$f(0)$ =____feet. – Height when just born. $f(20)$ =____feet. – Height at age of 20 years old.

$f(70)$ =____feet. – Height at age of 70 years old.

Answers: $f(0) = 1.5$ feet
$f(20) = 6.5$ feet
$f(70) = 6.5$ feet $f(90) = 5.5$ feet

$f(90)$ =____feet. – Height at age of
90 years old.

* Second lecture, Section 1.3 -- Linear and Quadratic Functions:

Find the explicit formulation of the age-height function: $f(x)$ =_____?

Answer:

$$f(x) = \begin{cases} \frac{1}{4}x + 1.5 & 0 \le x \le 20 \\ 6.5 & 20 < x \le 70 \\ 10 - \frac{1}{20}x & 70 < x \le 90 \end{cases}$$

Third lecture, Section 2.4 -The Derivative:

How fast does this person's height change? That is the DERIVATIVE of the age-height function, with unit of feet/year? $f'(x)$ =_____?

Answer:

$$f'(x) = \begin{cases} \frac{1}{4} & 0 \le x \le 20 \\ 0 & 20 < x \le 70 \\ -\frac{1}{20} & 70 < x \le 90 \end{cases}$$

Another Model of Height and Age:

$$g(x) = \begin{cases} -\frac{1}{180}(x-30)^2 + 6.5 & 0 \le x < 30 \\ 6.5 & 30 \le x < 70 \\ -\frac{1}{400}(x-70)^2 + 6.5 & 70 \le x \le 90 \end{cases}$$

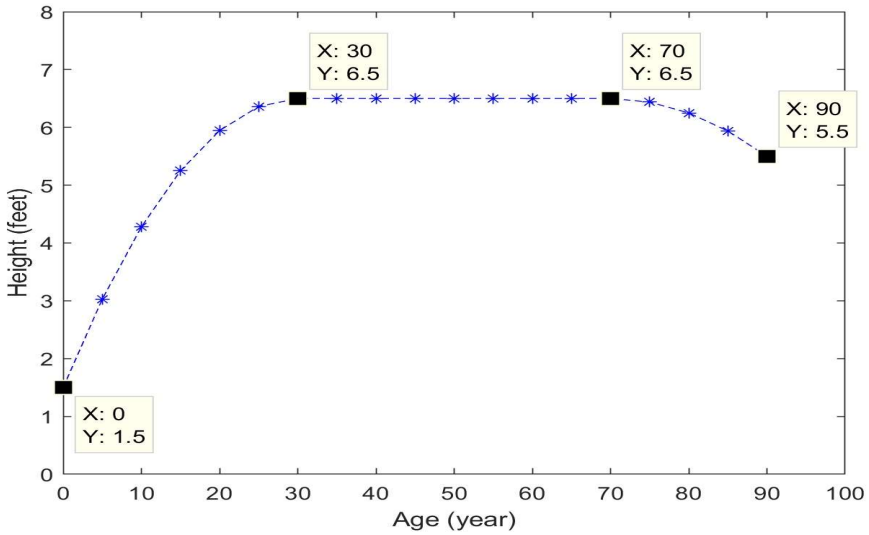

Figure 2: $g(x)$, Another Model of Height - Age

Questions :

If the function $g(x)$ is used to model the height-age relation,

1. Is the function $g(x)$ smooth (differentiable)?
2. How fast does this person's height change? What is the acceleration of the growth?
3. What are the advantages and disadvantages for each of the two models?